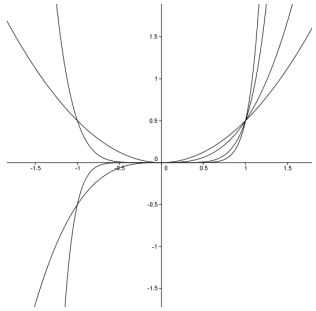
Hoofdstuk 9: Allerlei functies

9.1 Machtsfuncties en wortelfuncties

Opgave 1:

a.



b. (0,0) $(1, \frac{1}{2})$

c. y_1 en y_3

d. y_1 en y_3

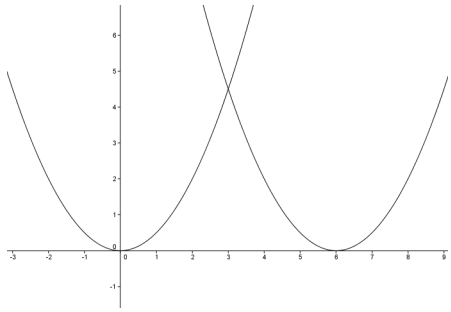
Opgave 2:

a. de grafiek van y_2 ontstaat uit die van y_1 door $T(0,2)$

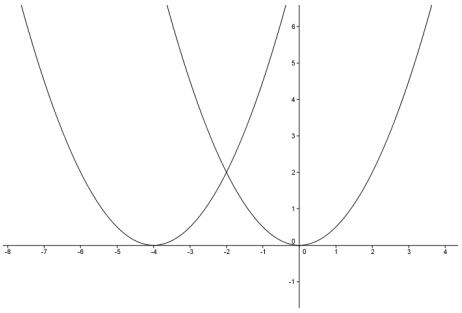
b. $y = \frac{1}{2}x^2 \xrightarrow{T(0,6)} y = \frac{1}{2}x^2 + 6$

Opgave 3:

a. $T(6,0)$



b. $T(-4,0)$



c. $y = \frac{1}{2}x^2 \xrightarrow{T(2,0)} y = \frac{1}{2}(x-2)^2$

Opgave 4:

a. $y = -5(x-2)^2 + 5$

b. $y = -5(x+3)^2 + 6$

c. $y = -5(x-7)^2$

Opgave 5:

g: $T(-2,0)$ dus $g(x) = 2(x+2)^2$

h: $T(2,-2)$ dus $h(x) = 2(x-2)^2 - 2$

k: $T(-1,-3)$ dus $k(x) = 2(x+1)^2 - 3$

l: $T(1,-4)$ dus $l(x) = 2(x-1)^2 - 4$

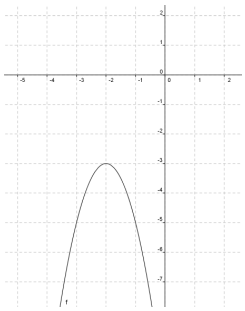
Opgave 6:

a. $y = -2x^2 \xrightarrow{T(-2,-3)} y = -2(x+2)^2 - 3$

bergparabool met top $(-2,-3)$

max $f(-2) = -3$

$B_f = \langle \leftarrow, -3]$

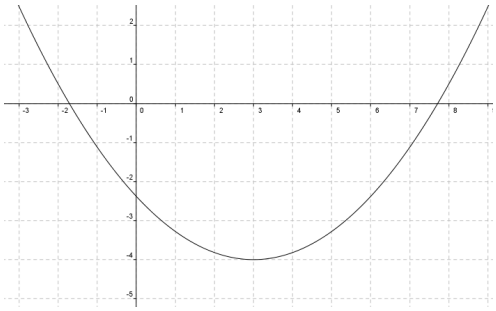


b. $y = 0{,}18x^2 \xrightarrow{T(3,-4)} y = 0{,}18(x-3)^2 - 4$

dalparabool met top $(3,-4)$

min $g(3) = -4$

$B_g = [-4, \rightarrow\rangle$



Opgave 7:

a. max $f(0) = 2$ $B_f = \langle \leftarrow, 2]$

b. max $g(2) = 8$ $B_g = \langle \leftarrow, 8]$

c. min $h(-1) = 0$ $B_h = [0, \rightarrow\rangle$

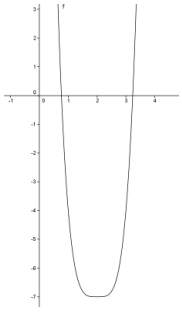
d. min $k(0) = 1$ $B_k = [1, \rightarrow\rangle$

e. max $l(100) = 0$ $B_l = \langle \leftarrow, 0]$

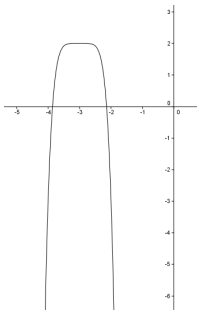
f. max $m(-0{,}1) = -0{,}3$ $B_m = \langle \leftarrow; -0{,}3]$

Opgave 8:

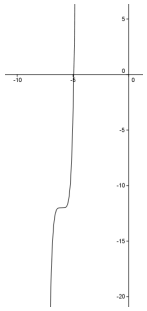
a. top $(2,-7)$



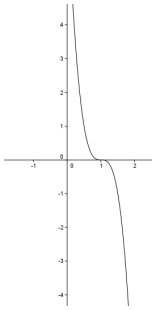
b. top $(-3,2)$



c. punt van symmetrie $(-6,-12)$

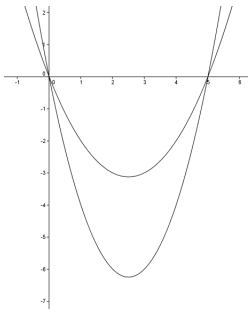


d. punt van symmetrie $(1,0)$

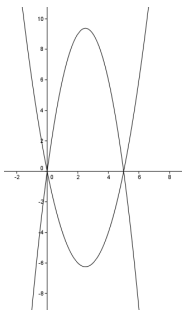


Opgave 9:

a. $y_1 \xrightarrow{V_{x-as,\frac{1}{2}}} y_2$



b. $y_1 \xrightarrow{V_{x-as,-1\frac{1}{2}}} y_3$



Opgave 10:

$y = -\frac{1}{2}x^3 \xrightarrow{T(-3,-5)} y = -\frac{1}{2}(x+3)^3 - 5 \xrightarrow{V_{x-as,-3}} y = 1\frac{1}{2}(x+3)^3 + 15$

Opgave 11:

a. $y = 0{,}3x^4 \xrightarrow{T(-5,6)} y = 0{,}3(x+5)^4 + 6 \xrightarrow{V_{x-as,3}} y = 0{,}9(x+5)^4 + 18$
top $(-5,18)$

b. $y = 0{,}3x^4 \xrightarrow{V_{x-as,3}} y = 0{,}9x^4 \xrightarrow{T(-5,6)} y = 0{,}9(x+5)^4 + 6$
top $(-5,6)$

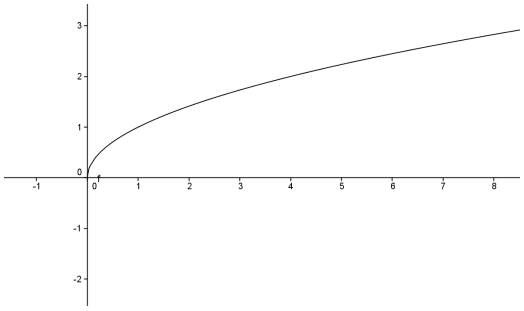
Opgave 12:

a. $V_{x-as,-1}$

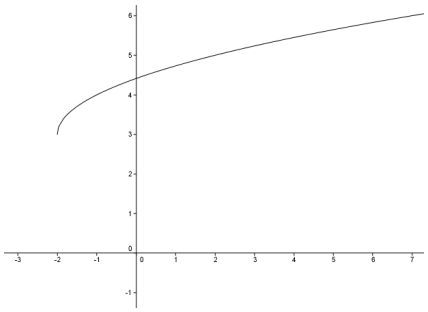
b. $y = -3(x-1)^2 + 6$

Opgave 13:

a. $D_f = [0,\rightarrow\rangle$
$B_f = [0,\rightarrow\rangle$



b. $T(-2,3)$



c. $T(1,-4)$

d. $y = \sqrt{x} \xrightarrow{V_{x-as,3}} y = 3\sqrt{x} \xrightarrow{T(1,0)} y = 3\sqrt{x-1}$

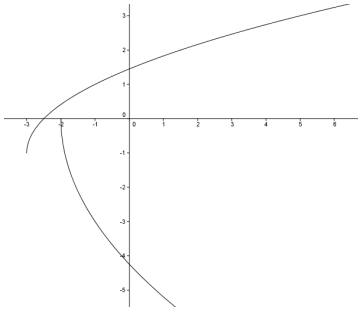
e. $V_{y-as,\frac{1}{2}}$

Opgave 14:

a. $y = \sqrt{x} \xrightarrow{V_{x-as,-3}} y = -3\sqrt{x} \xrightarrow{T(-2,0)} y = -3\sqrt{x+2}$

$y = \sqrt{x} \xrightarrow{T(-6,-1)} y = -1 + \sqrt{x+6} \xrightarrow{V_{y-as,\frac{1}{2}}} y = -1 + \sqrt{2x+6}$

b.



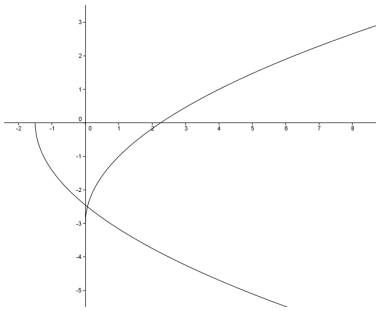
c. $D_f = [-2, \rightarrow\rangle$
$B_f = \langle\leftarrow, 0]$
$D_g = [-3, \rightarrow\rangle$
$B_g = [-1, \rightarrow\rangle$

Opgave 15:

a. $y = \sqrt{x} \xrightarrow{V_{x-as,2}} y = 2\sqrt{x} \xrightarrow{T(0,-3)} y = 2\sqrt{x} - 3$

$y = \sqrt{x} \xrightarrow{T(-6,0)} y = \sqrt{x+6} \xrightarrow{V_{y-as,\frac{1}{4}}} y = \sqrt{4x+6} \xrightarrow{V_{x-as,-1}} y = -\sqrt{4x+6}$

b.



c. $D_f = [0, \rightarrow\rangle$
$B_f = [-3, \rightarrow\rangle$
$D_g = [-1\frac{1}{2}, \rightarrow\rangle$
$B_g = \langle\leftarrow, 0]$

Opgave 16:

a. beginpunt $(-5,3)$ $D_f = [-5, \rightarrow\rangle$ $B_f = [3, \rightarrow\rangle$
b. beginpunt $(-1\frac{1}{2}, -7)$ $D_g = [-1\frac{1}{2}, \rightarrow\rangle$ $B_g = [-7, \rightarrow\rangle$
c. beginpunt $(-1,0)$ $D_h = [-1, \rightarrow\rangle$ $B_h = \langle\leftarrow, 0]$
d. beginpunt $(0,1)$ $D_k = [0, \rightarrow\rangle$ $B_k = [1, \rightarrow\rangle$
e. beginpunt $(1,-1)$ $D_l = [1, \rightarrow\rangle$ $B_l = \langle\leftarrow, -1]$
f. beginpunt $(0,-3)$ $D_m = [0, \rightarrow\rangle$ $B_m = [-3, \rightarrow\rangle$

Opgave 17:

a. $\sqrt{2x-5} = 3$ links en rechts kwadrateren
$2x - 5 = 9$
$2x = 14$
$x = 7$

b. de uitkomst van een wortel is groter of gelijk aan 0

Opgave 18:

a. $x = \sqrt{5x+14}$
$x^2 = 5x + 14$
$x^2 - 5x - 14 = 0$
$(x-7)(x+2) = 0$
$x = 7 \quad \vee \quad x = -2$ (vervalt)

b. $3x = \sqrt{8x + 20}$

$9x^2 = 8x + 20$

$9x^2 - 8x - 20 = 0$

$x = \frac{8 \pm \sqrt{784}}{18} = \frac{8 \pm 28}{18}$

$x = 2 \quad \vee \quad x = -\frac{10}{9}$ (vervalt)

c. $5\sqrt{x} = x$

$25x = x^2$

$-x^2 + 25x = 0$

$-x(x - 25) = 0$

$x = 0 \quad \vee \quad x = 25$

d. $3x = \sqrt{18x + 72}$

$9x^2 = 18x + 72$

$9x^2 - 18x - 72 = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4 \quad \vee \quad x = -2$ (vervalt)

Opgave 19:

a. $4 - 3\sqrt{x} = 2$

$-3\sqrt{x} = -2$

$\sqrt{x} = \frac{2}{3}$

$x = \frac{4}{9}$

b. $5\sqrt{x} - 2x = 0$

$5\sqrt{x} = 2x$

$25x = 4x^2$

$-4x^2 + 25x = 0$

$-4x(x - 6\frac{1}{4}) = 0$

$x = 0 \quad \vee \quad x = 6\frac{1}{4}$

c. $2x - 5\sqrt{x} = 3$

$2x - 3 = 5\sqrt{x}$

$4x^2 - 12x + 9 = 25x$

$4x^2 - 37x + 9 = 0$

$x = \frac{37 \pm \sqrt{1225}}{8} = \frac{37 \pm 35}{8}$

$x = 9 \quad \vee \quad x = \frac{1}{4}$ (vervalt)

d. $5x - 2\sqrt{x} = 3$

$5x - 3 = 2\sqrt{x}$

$25x^2 - 30x + 9 = 4x$

$25x^2 - 34x + 9 = 0$

$x = \dfrac{34 \pm \sqrt{256}}{50} = \dfrac{34 \pm 16}{50}$

$x = 1 \quad \vee \quad x = \frac{9}{25}$ (vervalt)

Opgave 20:

a. $2x + \sqrt{x} = 10$

$2x - 10 = -\sqrt{x}$

$4x^2 - 40x + 100 = x$

$4x^2 - 41x + 100 = 0$

$x = \dfrac{41 \pm \sqrt{81}}{8} = \dfrac{41 \pm 9}{8}$

$x = 4 \quad \vee \quad x = 6\frac{1}{4}$ (vervalt)

b. $\sqrt{x+12} = x$

$x + 12 = x^2$

$-x^2 + x + 12 = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4 \quad \vee \quad x = -3$ (vervalt)

c. $2x + \sqrt{x} = 6$

$2x - 6 = -\sqrt{x}$

$4x^2 - 24x + 36 = x$

$4x^2 - 25x + 36 = 0$

$x = \dfrac{25 \pm \sqrt{49}}{8} = \dfrac{25 \pm 7}{8}$

$x = 2\frac{1}{4} \quad \vee \quad x = 4$ (vervalt)

d. $10 - x\sqrt{x} = 2$

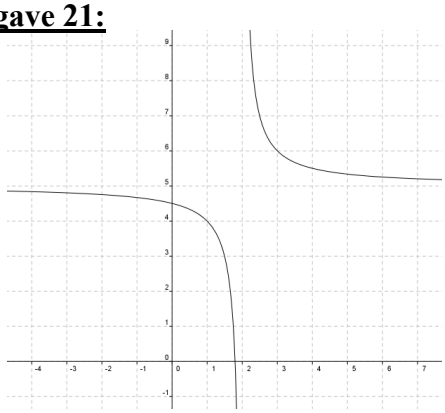
$-x\sqrt{x} = -8$

$x^3 = 64$

$x = 4$

9.2 Gebroken functies

Opgave 21:

a. 

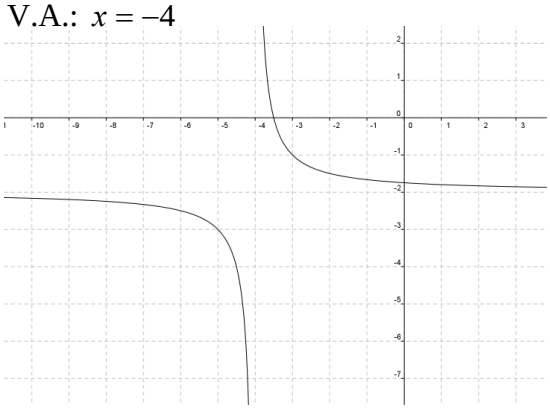
b. nee

c. nee

Opgave 22:

a. $T(-4,-2)$

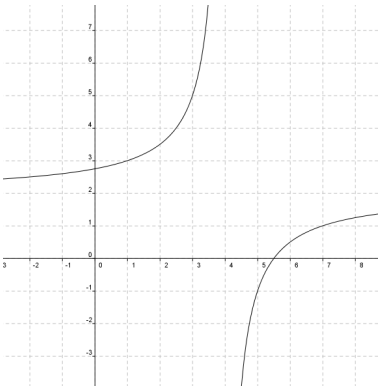
b. H.A.: $y = -2$
V.A.: $x = -4$

c. 

Opgave 23:

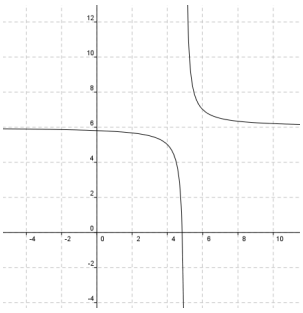
a. $y = \frac{1}{x} \xrightarrow{V_{x-as,-3}} y = \frac{-3}{x} \xrightarrow{T(4,2)} y = \frac{-3}{x-4} + 2$

b. H.A.: $y = 2$
V.A.: $x = 4$

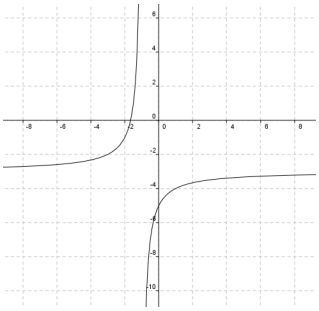


Opgave 24:

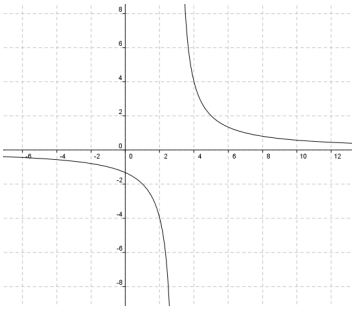
a. H.A.: $y = 6$
V.A.: $x = 5$



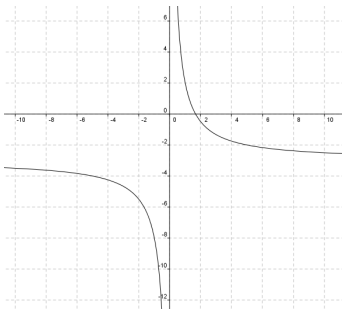
b. H.A.: $y = -3$
V.A.: $x = -1$



c. H.A.: $y = 0$
V.A.: $x = 3$

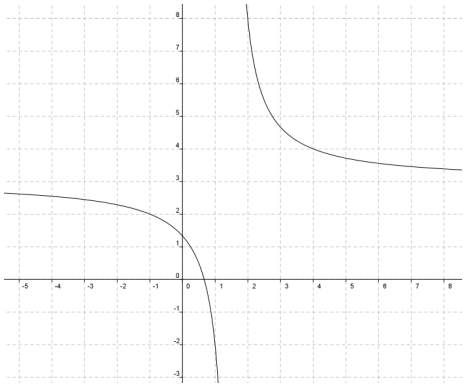


d. H.A.: $y = -3$
V.A.: $x = 0$



Opgave 25:

a.



b. $f(x)$ komt steeds dichter bij 3
H.A.: $y = 3$

c. $f(x)$ gaat naar $-\infty$
V.A.: $x = 1{,}5$

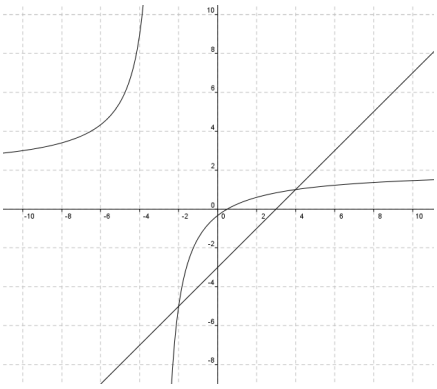
Opgave 26:

a. H.A.: $y = -1$
V.A.: $4 - x = 0$
Dus $x = 4$

b. H.A.: $y = 1$
V.A.: $5 + 2x = 0$
$2x = -5$
$x = -2\frac{1}{2}$

Opgave 27:

a.



b. $\dfrac{2x-1}{x+3} = x - 3$
$(x+3)(x-3) = 2x - 1$
$x^2 - 9 = 2x - 1$
$x^2 - 2x - 8 = 0$
$(x-4)(x+2) = 0$
$x = 4 \quad \vee \quad x = -2$
$-3 < x \leq -2 \quad \vee \quad x \geq 4$

Opgave 28:

a. $\dfrac{x+1}{x-1} = x - 1$
$(x-1)(x-1) = x + 1$
$x^2 - 2x + 1 = x + 1$
$x^2 - 3x = 0$
$x(x-3) = 0$
$x = 0 \quad \vee \quad x = 3$

b. $x = \dfrac{15}{x-2}$
$x(x-2) = 15$

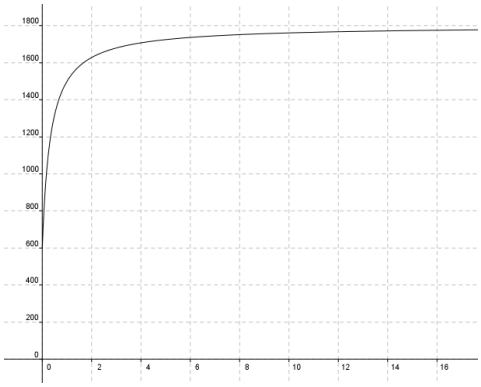
$x^2 - 2x - 15 = 0$
$(x-5)(x+3) = 0$
$x = 5 \quad \vee \quad x = -3$

c. $\dfrac{4x}{\sqrt{2x+6}} = \sqrt{2x+6}$
$4x = 2x + 6$
$2x = 6$
$x = 3$

d. $\dfrac{2x}{\sqrt{x^2-3}} = \sqrt{x^2-3}$
$x^2 - 3 = 2x$
$x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$x = 3 \quad \vee \quad x = -1$ (vervalt)

Opgave 29:

a. $N = 1800$
op den duur zijn er 1800 insecten

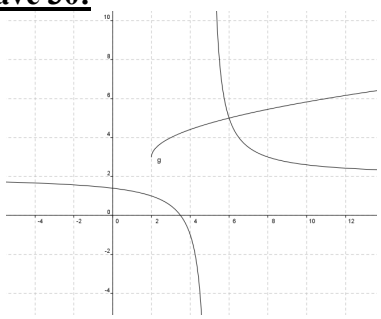
b. 

c. $y_1 = 1800 - \dfrac{1200}{1+3x}$ en $y_2 = 1760$ intersect geeft: $x = 9{,}7$
dus op de tiende dag

d. $N(4) - N(3) = 1708 - 1680 = 28$

e. $y_2 = 1680$ en $y_3 = 1745$
intersect y_1 en y_2 geeft $x = 3$
intersect y_1 en y_3 geeft $x = 6{,}9$
dus 4 dagen

Opgave 30:

a. 

b. $y_1 = \dfrac{2x-7}{x-5}$ en $y_2 = \sqrt{x-2}+3$ intersect geeft: $x = 6$

dus $2 \le x < 5 \quad \vee \quad x \ge 6$

Opgave 31:

a. $y_1 = \dfrac{3x-5}{x-2}$

$\left[\frac{dy}{dx}\right]_{x=4} = -0{,}25$

b. 2

c. 2 ; 0

Opgave 32:

$y_1 = \dfrac{3x-4}{2x-1}$ en $y_2 = nDeriv(y_1, x, x)$ en $y_3 = 0{,}2$

intersect van y_2 en y_3 geeft $x = -2 \quad \vee \quad x = 3$

dus $(-2,2)$ en $(3,1)$

Opgave 33:

$y_1 = \dfrac{2x-4}{x-4}$ en $y_2 = nDeriv(y_1, x, x)$ en $y_3 = -1$

intersect van y_2 en y_3 geeft $x = 2 \quad \vee \quad x = 6$

$y = -x + b$ door $(2,0)$ | $y = -x + b$ door $(6,4)$

$0 = -2 + b$ | $4 = -6 + b$

$b = 2$ | $b = 10$

$y = -x + 2$ | $y = -x + 10$

Opgave 34:

$rc = 0{,}6$

$y_1 = \dfrac{5x}{2x+3}$ en $y_2 = nDeriv(y_1, x, x)$ en $y_3 = 0{,}6$

intersect van y_2 en y_3 geeft $x = -4 \quad \vee \quad x = 1$

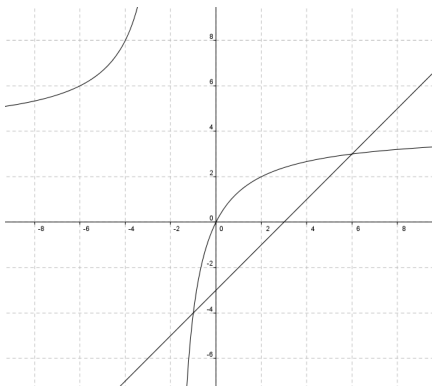
dus $(-4,4)$ en $(1,1)$

Opgave 35:

a. H.A.: $y = 4$

V.A.: $x + 2 = 0$

dus $x = -2$



b. $\dfrac{4x}{x+2} = x - 3$

$(x+2)(x-3) = 4x$

$x^2 - x - 6 = 4x$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6 \quad \vee \quad x = -1$

$x < -2 \quad \vee \quad -1 < x < 6$

c. $rc = 1$

$y_1 = \dfrac{4x}{x+2}$ en $y_2 = nDeriv(y_1, x, x)$ en $y_3 = 1$

intersect van y_2 en y_3 geeft $x = -4{,}83 \quad \vee \quad x = 0{,}83$

dus $(-4{,}83; 6{,}83)$ en $(0{,}83; 1{,}17)$

9.3 Logaritmische functies

Opgave 36:

a. $y_1 = 10$ en $y_2 = 2$ intersect geeft: $x = 0{,}30103$

b. $y_1 = 10$ en $y_2 = 5$ intersect geeft: $x = 0{,}69897$
dus $5 = 10^{0{,}69897}$

c. $y_1 = 10$ en $y_2 = 15$ intersect geeft: $x = 1{,}17609$
dus $15 = 10^{1{,}17609}$

Opgave 37:

a. $25 = 10^{\log 25}$

b. $\sqrt{2} = 10^{\log \sqrt{2}}$

Opgave 38:

a. $7 = 10^{\log 7}$
$7^x = (10^{\log 7})^x = 10^{x \cdot \log 7}$

b. $20 = 10^{\log 20}$

c. $7^x = 20$
$10^{x \cdot \log 7} = 10^{\log 20}$
$x \cdot \log 7 = \log 20$

d. $7^x = 20$ geeft $x \cdot \log 7 = \log 20$
dus $x = \dfrac{\log 20}{\log 7}$

Opgave 39:

a. ${}^2\log 80 = \dfrac{\log 80}{\log 2} = 6{,}322$

b. ${}^3\log 0{,}2 = \dfrac{\log 0{,}2}{\log 3} = -1{,}465$

c. ${}^5\log 50 = \dfrac{\log 50}{\log 5} = 2{,}431$

d. ${}^{\frac{1}{2}}\log 25 = \dfrac{\log 25}{\log \frac{1}{2}} = -4{,}644$

e. ${}^2\log 10 + {}^2\log 12 = \dfrac{\log 10}{\log 2} + \dfrac{\log 12}{\log 2} = 6{,}907$

f. ${}^{\frac{1}{3}}\log 20 - {}^{\frac{1}{3}}\log 10 = \dfrac{\log 20}{\log \frac{1}{3}} - \dfrac{\log 10}{\log \frac{1}{3}} = -0{,}631$

Opgave 40:

a. $2^{x+1} = (10^{\log 2})^{x+1} = (10^{0{,}301})^{x+1} = 10^{0{,}301x + 0{,}301}$

b. $3^{2x-1} = (10^{\log 3})^{2x-1} = (10^{0{,}477})^{2x-1} = 10^{0{,}954x - 0{,}477}$

c. $2 \cdot 5^x = 10^{\log 2} \cdot (10^{\log 5})^x = 10^{0{,}301} \cdot (10^{0{,}699})^x = 10^{0{,}301} \cdot 10^{0{,}699x} = 10^{0{,}699x + 0{,}301}$

Opgave 41:

a. $6^{2x+1} = 6^1 \cdot 6^{2x} = 6 \cdot (10^{\log 6})^{2x} = 6 \cdot (10^{0,778})^{2x} = 6 \cdot 10^{1,556x}$

b. $(\frac{1}{2})^{3x-2} = (\frac{1}{2})^{-2} \cdot (\frac{1}{2})^{3x} = 4 \cdot (10^{\log \frac{1}{2}})^{3x} = 4 \cdot (10^{-0,301})^{3x} = 4 \cdot 10^{-0,903x}$

c. $1{,}18^{4x-1} = 1{,}18^{-1} \cdot 1{,}18^{4x} = 0{,}847 \cdot (10^{\log 1,18})^{4x} = 0{,}847 \cdot (10^{0,072})^{4x} = 0{,}847 \cdot 10^{0,288x}$

Opgave 42:

a. $N(10) - N(9) = 165 - 146 = 19$
$N(40) - N(39) = 1612 - 1571 = 41$

b. $N = 2000$
op den duur zijn er 2000 ratten

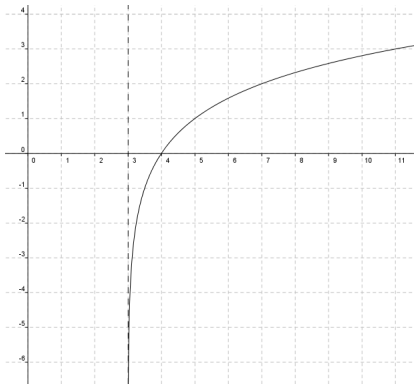
c. $y_1 = \dfrac{2000}{1 + 40 \cdot 0{,}88^x}$ en $y_2 = nDeriv(y_1, x, x)$
het maximum van y_2 is als $x = 28{,}9$ en dan is $y = 63{,}9$
dus op $t = 28{,}9$ komen er 64 ratten per dag bij

d. $G = 2000$ en $a = 40$
$0{,}88^t = (10^{\log 0,88})^t = (10^{-0,056})^t = 10^{-0,056t}$
$b = -0{,}056$

e. $40 = 10^{\log 40} = 10^{1,602}$
dus $c = -0{,}056$ en $d = 1{,}602$

Opgave 43:

a. $x = 3$

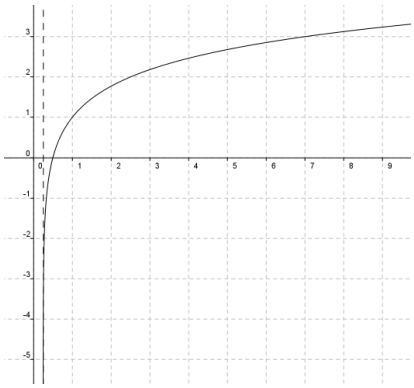
b. 

c. $^2\log(x-3) = \dfrac{\log(x-3)}{\log 2}$

Opgave 44:

a. V.A.: $4x - 1 = 0$
$4x = 1$
$x = \frac{1}{4}$

b. $^3\log(4x-1) = 2$
$4x - 1 = 3^2$
$4x - 1 = 9$
$4x = 10$
$x = 2{,}5$



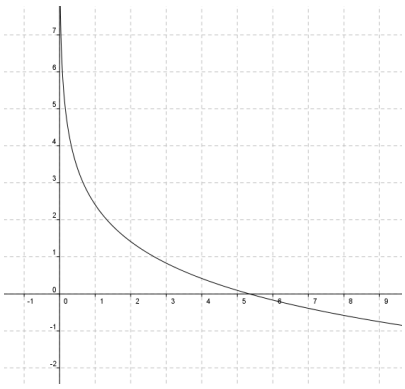
$\frac{1}{4} < x \le 2{,}5$

Opgave 45:

a. V.A.: $3x = 0$
$x = 0$

b. $4 + {}^{\frac{1}{2}}\log 3x = 0$
${}^{\frac{1}{2}}\log 3x = -4$
$3x = (\frac{1}{2})^{-4}$
$3x = 16$
$x = 5\frac{1}{3}$
$0 < x \le 5\frac{1}{3}$

c. $g(2{,}5) = 1{,}09$
$g(x) \le 1{,}09$

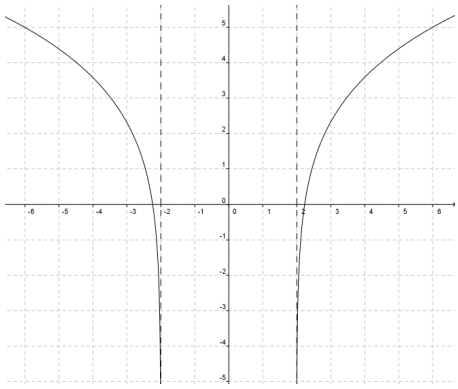


Opgave 46:

a. V.A.: $x^2 - 4 = 0$
$x^2 = 4$
$x = 2 \quad \vee \quad x = -2$

b. ${}^{2}\log(x^2 - 4) = 3$
$x^2 - 4 = 2^3$
$x^2 - 4 = 8$
$x^2 = 12$
$x = \sqrt{12} = 3{,}46 \quad \vee \quad x = -\sqrt{12} = -3{,}46$
$-3{,}46 \le x < -2 \quad \vee \quad 2 < x \le 3{,}46$

c. $f(x)$ neemt alle waarden aan

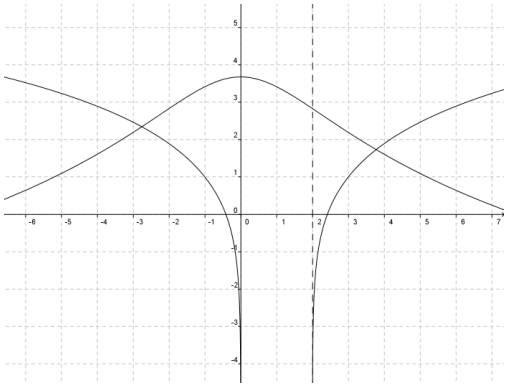


Opgave 47:

a. f heeft geen V.A.
g: V.A.: $x^2 - 2x = 0$
$x(x-2) = 0$
$x = 0 \quad \vee \quad x = 2$

b. twee snijpunten
$y_1 = 6 + \dfrac{\log(x^2+5)}{\log(\frac{1}{2})}$
$y_2 = \dfrac{\log(x^2 - 2x)}{\log 3}$
intersect geeft: $(-2{,}759; 2{,}344)$ en $(3{,}776; 1{,}732)$

c. $-2{,}759 < x < 0 \quad \vee \quad 2 < x < 3{,}776$



Opgave 48:

a. $D(0) = -5{,}88$ dus 5,88 m diep
$D(500) = -1{,}88$ dus 1,88 m diep

b. $y_1 = -9{,}6 + \dfrac{\log(x^2 + 400)}{\log 5}$ en $y_2 = -2{,}5$ intersect geeft $x = 302$

dus op 302 m

c. ja

d. als je $a = 500$ invult moet er $D = -2{,}3$ uitkomen

D blijft onveranderd, dus je moet *a* met een getal vermenigvuldigen

e. $a = 500$

$D = -9{,}6 + {}^5\log((\frac{500}{p})^2 + 400)$

$y_1 = -9{,}6 + \dfrac{\log((\frac{500}{x})^2 + 400)}{\log 5}$ en $y_2 = -2{,}3$ intersect geeft $x = 1{,}41$

dus $p = 1{,}41$

9.4 Transformaties en formules

Opgave 49:

a. $f(x) = 2x^2 - 3x \xrightarrow{T(1,2)} g(x) = 2(x-1)^2 - 3(x-1) + 2$

$g(x) = 2(x-1)^2 - 3(x-1) + 2$

$= 2(x^2 - 2x + 1) - 3x + 3 + 2$

$= 2x^2 - 4x + 2 - 3x + 3 + 2$

$= 2x^2 - 7x + 7$

b. $f(x) = 2x^2 - 3x \xrightarrow{V_{y-as,\frac{1}{4}}} h(x) = 2(4x)^2 - 3 \cdot 4x$

$h(x) = 2(4x)^2 - 3 \cdot 4x$

$= 2 \cdot 16x^2 - 12x$

$= 32x^2 - 12x$

Opgave 50:

a. $f(x) = x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0 \quad \vee \quad x^2 = 4$

$x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -2$

b. als je vermenigvuldigt t.o.v. de *y*-as met factor 3 , worden de *x*-coördinaten vermenigvuldigd met 3

dus $x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -2$ wordt: $x = 0 \quad \vee \quad x = 6 \quad \vee \quad x = -6$

Opgave 51:

a. $f(x) = x^4 - 3x \xrightarrow{V_{y-as,2}} g(x) = (\frac{1}{2}x)^4 - 3 \cdot \frac{1}{2}x$

$g(x) = (\frac{1}{2}x)^4 - 3 \cdot \frac{1}{2}x$

$= \frac{1}{16}x^4 - 1\frac{1}{2}x$

b. $h(x) = 4x^3 - 2x^2 \xrightarrow{V_{y-as,-\frac{1}{3}}} k(x) = 4 \cdot (-3x)^3 - 2 \cdot (-3x)^2$

$k(x) = 4 \cdot (-3x)^3 - 2 \cdot (-3x)^2$

$= 4 \cdot -27x^3 - 2 \cdot 9x^2$

$= -108x^3 - 18x^2$

Opgave 52:

a. $x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x = 0 \quad \vee \quad x^2 = 9$

$x = 0 \quad x = 3 \quad \vee \quad x = -3$

b. $f(x) = x^3 - 9x \xrightarrow{V_{y-as,4}} g(x) = (\frac{1}{4}x)^3 - 9 \cdot \frac{1}{4}x$

$g(x) = (\frac{1}{4}x)^3 - 9 \cdot \frac{1}{4}x$

$= \frac{1}{64}x^3 - \frac{9}{4}x$

c. $\frac{1}{64}x^3 - \frac{9}{4}x = 0$

$x^3 - 144x = 0$

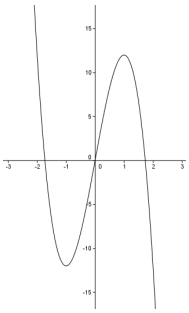
$x(x^2 - 144) = 0$

$x = 0 \quad \vee \quad x^2 = 144$

$x = 0 \quad \vee \quad x = 12 \quad \vee \quad x = -12$

d. als je de nulpunten van f vermenigvuldigt met 4 krijg je de nulpunten van g

Opgave 53:



a. $f'(x) = -18x^2 + 18$

b. $-18x^2 + 18 = 0$

$-18x^2 = -18$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$

min $f(-1) = -12$

max $f(1) = 12$

c. $f(x) = -6x^3 + 18x \xrightarrow{V_{y-as,4}} g(x) = -6 \cdot (\frac{1}{4}x)^3 + 18 \cdot \frac{1}{4}x$

$g(x) = -6 \cdot (\frac{1}{4}x)^3 + 18 \cdot \frac{1}{4}x$

$= -6 \cdot \frac{1}{64}x^3 + 4\frac{1}{2}x$

$= -\frac{3}{32}x^3 + 4\frac{1}{2}x$

d. $g'(x) = -\frac{9}{32}x^2 + 4\frac{1}{2} = 0$

$-9x^2 + 144 = 0$

$-9x^2 = -144$

$x^2 = 16$

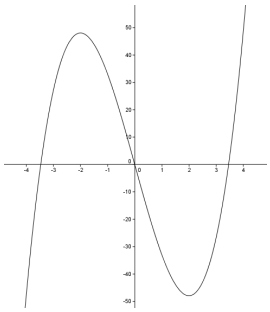
$x = 4 \quad \vee \quad x = -4$

min $g(-4) = -12$

max $g(4) = 12$

e. als je de x-coördinaten van de toppen van de grafiek van f vermenigvuldigt met 4 krijg je de x-coördinaten van de toppen van de grafiek van g.

Opgave 54:



a. $f(x) = 3x^3 - 36x = 0$

$3x(x^2 - 12) = 0$

$x = 0 \quad \vee \quad x^2 = 12$

$x = 0 \quad \vee \quad x = \sqrt{12} \quad \vee \quad x = -\sqrt{12}$

$f'(x) = 9x^2 - 36 = 0$

$9x^2 = 36$

$x^2 = 4$

$x = 2 \quad \vee \quad x = -2$

max $f(-2) = 48$

min $f(2) = -48$

b. $f(x) = 3x^3 - 36x \xrightarrow{V_{y-as,3}} g(x) = 3 \cdot (\frac{1}{3}x)^3 - 36 \cdot \frac{1}{3}x$

$g(x) = 3 \cdot (\frac{1}{3}x)^3 - 36 \cdot \frac{1}{3}x$

$= 3 \cdot \frac{1}{27}x^3 - 12x$

$= \frac{1}{9}x^3 - 12x$

c. de nulpunten worden vermenigvuldigd met 3 dus $x = 0 \quad \vee \quad x = 3\sqrt{12} \quad \vee \quad x = -3\sqrt{12}$
extreme waarden: de x-coördinaten worden vermenigvuldigd met 3 dus:
max $g(-6) = 48$
min $g(6) = -48$

Opgave 55:

a. $g(x) = f(x+3)$
$= (x+3)^2 + 3 \cdot (x+3)$
$= x^2 + 6x + 9 + 3x + 9$
$= x^2 + 9x + 18$

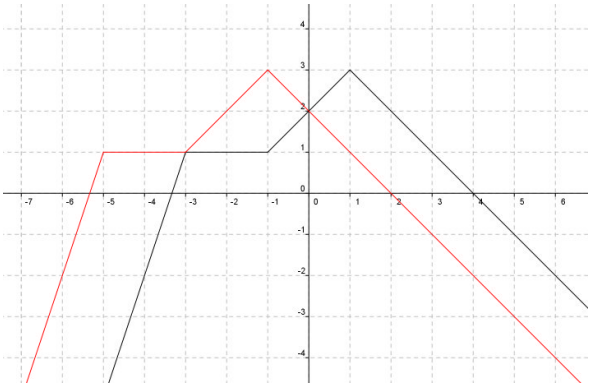
b. $h(x) = 3 \cdot f(x)$
$= 3 \cdot (x^2 + 3x)$
$= 3x^2 + 9x$

c. $j(x) = f(3x)$
$= (3x)^2 + 3 \cdot 3x$
$= 9x^2 + 9x$

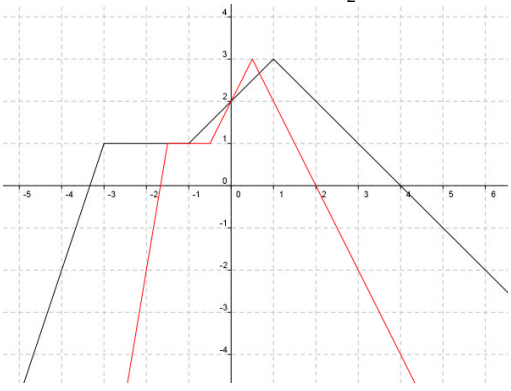
d. $k(x) = f(x) + 3$
$= x^2 + 3x + 3$

Opgave 56:

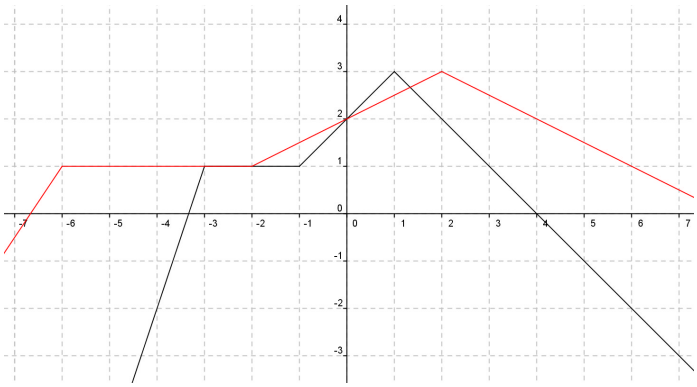
a. $y = f(x) \xrightarrow{T(-2,0)} y = f(x+2)$



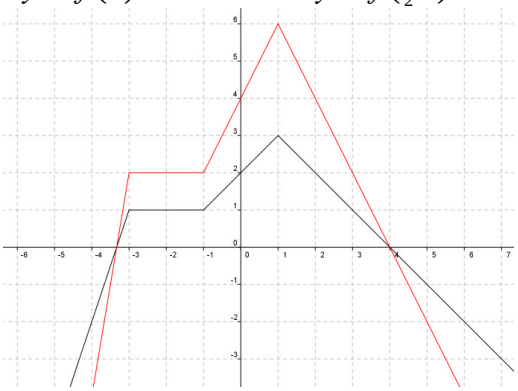
b. $y = f(x) \xrightarrow{V_{x-as,\frac{1}{2}}} y = \frac{1}{2} \cdot f(x)$



c. $y = f(x) \xrightarrow{V_{x-as,2}} y = 2 \cdot f(x)$



d. $y = f(x) \xrightarrow{V_{y-as,2}} y = f(\frac{1}{2}x)$



Opgave 57:

a. $f(10) = 0{,}2 \cdot 10^2 = 20$

b. $10p = 30$

$p = 3$

c. $f(x) = 0{,}2x^2 \xrightarrow{V_{y-as,3}} g(x) = 0{,}2 \cdot (\frac{1}{3}x)^2$

$g(x) = 0{,}2 \cdot (\frac{1}{3}x)^2$

$= \frac{1}{5} \cdot \frac{1}{9}x^2$

$= \frac{1}{45}x^2$

d. $20q = 40$

$q = 2$

e. $f(x) = 0{,}2x^2 \xrightarrow{V_{x-as,2}} h(x) = 2 \cdot 0{,}2x^2$

$h(x) = 2 \cdot 0{,}2x^2$

$= 0{,}4x^2$

Opgave 58:

a. de grafiek van f gaat door het punt $(2,5)$

dus $5p = 15$

dus $p = 3$

b. $f(x) = 1 + 2^x \xrightarrow{V_{x-as,3}} g(x) = 3 + 3 \cdot 2^x$

c. H.A.: $y = 3$

d. de grafiek van f gaat door het punt $(x,17)$

$1+2^x=17$

$2^x=16$

$2^x=2^4$

$x=4$

$4q=-1$

$q=-\frac{1}{4}$

e. $f(x)=1+2^x \xrightarrow{V_{y-as,-\frac{1}{4}}} h(x)=1+2^{-4x}$

$h(x)=1+2^{-4x}$

$=1+(2^{-4})^x$

$=1+\left(\frac{1}{2^4}\right)^x$

$=1+(\frac{1}{16})^x$

Opgave 59:

a. de grafiek van f gaat door het punt $(x,-2)$

$-2+{}^3\log(x-1)=-2$

${}^3\log(x-1)=0$

$x-1=3^0=1$

$x=2$

dus $2a=-10$

$a=-5$

b. $f(x)=-2+{}^3\log(x-1) \xrightarrow{V_{y-as,-5}} g(x)=-2+{}^3\log(-\frac{1}{5}x-1)$

c. V.A.: $-\frac{1}{5}x-1=0$

$-\frac{1}{5}x=1$

$x=-5$

Opgave 60:

a. $B(10)=1000\cdot 1{,}05^{10}=1628{,}89$

$B(20)=1000\cdot 1{,}05^{20}=2653{,}30$

b. 10 jaar is 1 tiental jaren

c. $T=\frac{1}{10}t$ dus $t=10T$

dus $B=1000\cdot 1{,}05^t=1000\cdot 1{,}05^{10T}$

Opgave 61:

a. $w=\frac{1}{7}t$ dus $t=7w$

$N=500\cdot 1{,}075^t$

$=500\cdot 1{,}075^{7w}$

$=500\cdot(1{,}075^7)^w$

$=500\cdot 1{,}659^w$

b. $t=\frac{1}{24}u$

$N=500\cdot 1{,}075^t$

$= 500 \cdot 1{,}075^{\frac{1}{24}u}$

$= 500 \cdot (1{,}075^{\frac{1}{24}})^{u}$

$= 500 \cdot 1{,}003^{u}$

Opgave 62:

a. $g_{dag} = 1{,}075$ dus 7,5% toename

b. $g_{week} = 1{,}659$ dus 65,9% toename

$g_{uur} = 1{,}003$ dus 0,3% toename

Opgave 63:

a. $N' = 960t - 120t^2 = 0$

$120t(8-t) = 0$

$t = 0 \quad \vee \quad t = 8$

$N(8) = 10240$ om 17.00 uur

b. $k = 4t$ dus $t = \frac{1}{4}k$

$N = 480t^2 - 40t^3$

$= 480 \cdot (\frac{1}{4}k)^2 - 40 \cdot (\frac{1}{4}k)^3$

$= 480 \cdot \frac{1}{16}k^2 - 40 \cdot \frac{1}{64}k^3$

$= 30k^2 - \frac{5}{8}k^3$

c. $t = 1{,}75$ geeft $N = 1256$

$t = 1{,}25$ geeft $N = 672$

dus een toename van $1256 - 672 = 584$

d. $N'(k) = 60k - 1{,}875k^2$

$N'(9) = 388$

Opgave 64:

$V_{y-as,4}$

9.5 Formules omwerken

Opgave 65:

$y = \frac{2}{x}$

$xy = 2$

$x = \frac{2}{y}$

Opgave 66:

a. $A = \frac{B}{B+2}$

$A(B+2) = B$

$AB + 2A = B$

$AB - B = -2A$

$B(A-1) = -2A$

$B = \frac{-2A}{A-1}$ of $B = \frac{2A}{1-A}$

b. $P = \frac{Q-5}{Q}$

$PQ = Q - 5$

$PQ - Q = -5$

$Q(P-1) = -5$

$Q = \frac{-5}{P-1}$ of $Q = \frac{5}{1-P}$

c. $R = \frac{F-2}{F-1}$

$R(F-1) = F - 2$

$RF - R = F - 2$

$RF - F = R - 2$

$F(R-1) = R - 2$

$F = \frac{R-2}{R-1}$

Opgave 67:

a. $p = 0{,}6$

$K = 10492{,}5$ dus 10492,50 euro

b. $p = 0{,}95$

$K = 83905$ dus 83905 euro

c. 100% dus $p = 1$ maar dat mag je niet invullen in de formule want dan wordt de noemer nul

d. $K = \frac{4200 - 5p}{1-p}$

$K(1-p) = 4200 - 5p$

$K - Kp = 4200 - 5p$

$5p - Kp = 4200 - K$

$p(5 - K) = 4200 - K$

$p = \dfrac{4200 - K}{5 - K}$

e. $p = 0{,}85$ dus 85%

Opgave 68:

a. $\dfrac{1}{a} = 2 + \dfrac{1}{b} = \dfrac{2b}{b} + \dfrac{1}{b} = \dfrac{2b+1}{b}$

b. $a(2b+1) = b$

$a = \dfrac{b}{2b+1}$

c. $\dfrac{1}{a} = 2 + \dfrac{1}{b}$

$\dfrac{1}{a} - 2 = \dfrac{1}{b}$

$\dfrac{1}{a} - \dfrac{2a}{a} = \dfrac{1}{b}$

$\dfrac{1-2a}{a} = \dfrac{1}{b}$

$b = \dfrac{a}{1-2a}$

Opgave 69:

a. $\dfrac{1}{p} = 5 - \dfrac{2}{q}$

$\dfrac{1}{p} = \dfrac{5q}{q} - \dfrac{2}{q}$

$\dfrac{1}{p} = \dfrac{5q-2}{q}$

$p = \dfrac{q}{5q-2}$

b. $\dfrac{1}{m} = \dfrac{1}{2} - \dfrac{3}{n}$

$\dfrac{3}{n} = \dfrac{1}{2} - \dfrac{1}{m}$

$\dfrac{3}{n} = \dfrac{m}{2m} - \dfrac{2}{2m}$

$\dfrac{3}{n} = \dfrac{m-2}{2m}$

$\dfrac{n}{3} = \dfrac{2m}{m-2}$

$n = \dfrac{6m}{m-2}$

Opgave 70:

a. $F = \frac{1}{K} + \frac{1}{2K}$

$F = \frac{2}{2K} + \frac{1}{2K}$

$F = \frac{3}{2K}$

$K = \frac{3}{2F}$

b. $N = \frac{2R+2}{5R+2}$

$N(5R+2) = 2R+2$

$5NR + 2N = 2R + 2$

$5NR - 2R = 2 - 2N$

$R(5N-2) = 2-2N$

$R = \frac{2-2N}{5N-2}$

c. $\frac{1}{T} = 10 - \frac{2}{S}$

$\frac{1}{T} = \frac{10S}{S} - \frac{2}{S}$

$\frac{1}{T} = \frac{10S-2}{S}$

$T = \frac{S}{10S-2}$

d. $\frac{6}{B} = \frac{5}{8} - \frac{2}{A}$

$\frac{2}{A} = \frac{5}{8} - \frac{6}{B}$

$\frac{2}{A} = \frac{5B}{8B} - \frac{48}{8B}$

$\frac{2}{A} = \frac{5B-48}{8B}$

$\frac{A}{2} = \frac{8B}{5B-48}$

$A = \frac{16B}{5B-48}$

Opgave 71:

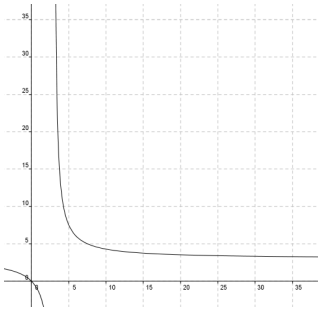
a. $\frac{1}{3} = \frac{1}{b} + \frac{1}{v}$

$\frac{1}{3} - \frac{1}{v} = \frac{1}{b}$

$\frac{v}{3v} - \frac{3}{3v} = \frac{1}{b}$

$$\frac{v-3}{3v}=\frac{1}{b}$$

$$b=\frac{3v}{v-3}$$



b. H.A.: $b=3$
V.A.: $v=3$
H.A.: als het voorwerp ver weg staat is de beeldafstand 3 cm
V.A.: als $v=3$ is er geen beeld

c. $v=b$

$$v=\frac{3v}{v-3}$$

$$v(v-3)=3v$$

$$v^2-3v=3v$$

$$v^2-6v=0$$

$$v(v-6)=0$$

$$v=0 \quad \vee \quad v=6$$

dus als $v=6$ cm

d. $\frac{b}{v}=2$

$$b=2v$$

$$2v=\frac{3v}{v-3}$$

$$2v(v-3)=3v$$

$$2v^2-6v=3v$$

$$2v^2-9v=0$$

$$2v(v-4\tfrac{1}{2})=0$$

$$v=0 \quad \vee \quad v=4\tfrac{1}{2}$$

dus als $v=4\frac{1}{2}$ cm

Opgave 72:

a. $y=\log(2x+1)$

$$y=^{10}\log(2x+1)$$

$$2x+1=10^y$$

b. $2x=-1+10^y$

$$x=-\tfrac{1}{2}+\tfrac{1}{2}\cdot 10^y$$

Opgave 73:

a. $\frac{1}{2}\cdot 10^{A-3}\neq 5^{A-3}$

b. $\frac{1}{2}\cdot 10^{A-3}=\frac{1}{2}\cdot 10^{A}\cdot 10^{-3}=0{,}0005\cdot 10^{A}$

Opgave 74:

a. $N=\log(5P+2)$

$$5P+2=10^N$$

$5P = -2 + 10^N$

$P = -\frac{2}{5} + \frac{1}{5} \cdot 10^N$

b. $F = 5\log(N) - 8$

$F + 8 = 5\log(N)$

$\frac{1}{5}F + \frac{8}{5} = \log(N)$

$N = 10^{\frac{1}{5}F + \frac{8}{5}}$

c. $0{,}5D = \log(4Q + 1) - 2$

$0{,}5D + 2 = \log(4Q + 1)$

$4Q + 1 = 10^{0{,}5D+2}$

$4Q = -1 + 10^{0{,}5D+2}$

$Q = -\frac{1}{4} + \frac{1}{4} \cdot 10^{0{,}5D+2}$

Opgave 75:

a. $A = 2\log(B) - 4$

$A + 4 = 2\log(B)$

$\frac{1}{2}A + 2 = \log(B)$

$B = 10^{\frac{1}{2}A+2}$

b. $B = 10^{\frac{1}{2}A+2} = 10^{\frac{1}{2}A} \cdot 10^2 = 100 \cdot 10^{\frac{1}{2}A}$

c. $B = 100 \cdot 10^{\frac{1}{2}A} = 100 \cdot (10^{\frac{1}{2}})^A = 100 \cdot 3{,}16^A$

Opgave 76:

a. $R = 2\log(s) - 6$

$R + 6 = 2\log(s)$

$\frac{1}{2}R + 3 = \log(s)$

$s = 10^{\frac{1}{2}R+3} = 10^3 \cdot 10^{\frac{1}{2}R} = 1000 \cdot (10^{\frac{1}{2}})^R = 1000 \cdot 3{,}16^R$

b. $5K = 3\log(N) + 2$

$5K - 2 = 3\log(N)$

$\frac{5}{3}K - \frac{2}{3} = \log(N)$

$N = 10^{\frac{5}{3}K-\frac{2}{3}} = 10^{-\frac{2}{3}} \cdot 10^{\frac{5}{3}K} = 0{,}22 \cdot (10^{\frac{5}{3}})^K = 0{,}22 \cdot 46{,}42^K$